An Inquiry into the Relationship Between the Probability Language Skills and Achievement

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Abstract

The cognitive processes underlying mathematical thinking are intrinsically tied to language, underscoring the interdependence of linguistic capabilities and mathematical proficiency. This study employed a correlational research design to examine the probability language skills of secondary school students and to explore how these skills correlate with their probability achievement. A sample of N = 49 students participated in probability language assessments alongside classroom probability achievement tests. Results revealed that while students demonstrated strong probability-related linguistic abilities, their comprehension and interpretation of probability concepts were notably low. Furthermore, a significant positive correlation [r (49) = 0.532, p < .01] was observed between students' probability language skills and their performance in the achievement tests. These findings underscore the potential role of mathematical language in the cognitive assimilation of mathematical concepts.

Keywords: *mathematical language, mathematics symbols, mathematical syntax, mathematics vocabulary, probability*

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1. Introduction

Imagine a teacher guiding students to draw a table and explaining that it must have 5 columns and 5 rows. Some learners begin wondering how a column used in buildings is used to draw a table, let alone having rows and not legs. To add to the confusion, there are signs such as +, – , and = that are commonly used in mathematics and other disciplines. Whenever such words and symbols are used, they may communicate a mathematical process or concept (Makonye, 2019), especially in a mathematical expression or statement. Many other signs are used in mathematics and may vary depending on the topic. To grasp the meaning of what is communicated in mathematical texts, symbols, or diagrams, one needs to understand the meanings of the symbols and expressions (Njiku, 2024), since mathematics is analytical in nature. Understanding mathematical symbols and vocabulary has been an area of research interest (Jourdain & Sharma, 2016;

Sepeng & Madzorera, 2014). Learners have been found to misinterpret the meanings communicated mathematically using various symbols, vocabulary, and diagrams (Chin et al., 2022; Sepeng & Madzorera, 2014). For example, Thompson and Chappell (2007) show that even simple operations such as + and – may be problematic. For instance, when simplifying the expression 12 – 4 + 5, students incorrectly wrote 12 – 4 = 8 + 5 = 13, implying that 12 – 4 = 13. Mathematical language has its semiotics. For example, Chin et al. (2022) show that the wording of the expression 2 + 2 x 3, when read left to right, confuses learners, because the operation 2 x 3 must be performed first, whereas 2 + appears first. These difficulties explain the need to draw attention to learners' difficulties and give more attention to mathematical language in general for effective learning.

Njiku (2024) explains that mathematics language is a foreign language to many, which is learnt and spoken at school. It is only at school that students have an opportunity to communicate mathematically, including speaking, writing, reading, and listening to mathematics language (Thompson & Chappell, 2007). However, at home, it may involve naming small numbers and informal expressions, but it also points out that such uses do not directly or correctly map onto the mathematical syntax. When learners discuss mathematics concepts, they tend to use peer-appropriate everyday language, which may inhibit the development of their understanding of the required mathematical register (Kotsopoulos, 2007). Nevertheless, in everyday conversations, learners' discussions may reveal complex mathematical structures, though not in a formal way, requiring teachers to bridge the gap between everyday and formal mathematical language (Simpson & Cole, 2015). Multiple representations may be used to effectively communicate mathematical meaning (Arias de Sanchez, 2022), and articulation involves verbal embodiment of mathematical symbolism, expressions, or equations (Chin et al., 2022). This communication involves, for example, semantically representing an algebraic expression graphically or syntactically, whereby mixed fractions may be presented as improper fractions. This paper examines whether competences in mathematical language relate to performance in probability.

In probability, a common topic in school curricula, language problems have been observed (Groth et al., 2016). Whereas probability as a topic shares much with everyday language, its formal mathematical meaning may still pose difficulties for learning probability. In everyday language, probability is used in discussions of weather forecasts, the possibility of a traffic jam, and political debates about which candidate is favoured. Everyday discussions about probabilities may

involve informal vocabulary different from formal probability language but useful for learning (Findley et al., 2021). Students have been reported to struggle with simple words such as likely, unlikely, average, and random (Groth et al., 2016), especially because of their imprecise use in everyday communication (Findley et al., 2021). These difficulties may influence students' performance in probability, and this study examines students' competencies in probability language and their relationship with performance.

2. Theoretical Framework

Language is not only a way by which people communicate, but it also aids thinking. This notion is found in classical work by Vygotsky (1986), and other scholars such as John Dewey and Paulo Freire have adopted this view of the importance of language as a socio-cultural element informing learning to a democratic notion of learning and critical approaches to education, respectively (Aubrey & Riley, 2022). Language influences thought, and since mathematics is a thought-based discipline, this renders the topic necessary, especially for mathematics instruction. Language is context-based (Arias de Sanchez, 2022; Vygotsky, 1986), and this context may influence how it accommodates mathematical aspects. Kotsopoulos (2007), for example, shows that everyday language influences students' approaches to mathematics, since mathematics is language-dependent. The interaction between the mathematical language on which mathematics depends and everyday language creates difficulties in learning the subject. Mathematical language possesses a register whose meaning is shared by the social group of mathematicians or those who do mathematics. Although Vygotsky's theory argues that language comprehension does not necessarily require understanding specific vocabulary, mathematics is more analytical than common language, whose vocabulary may require more attention. Appropriate conceptions of probability or statistical terms are important and should be reinforced, especially when students are required to construct and articulate viable arguments and conjectures (Simpson & Cole, 2015). For instance, Nabbout-Cheiban (2017) found that students misinterpreted the concept of independent events as a property of a pair of events; instead, they considered them as one-way, as in the case of one person who may depend on another, where the latter is independent of the former. The use of the words 'dependent' and 'independent' in daily life made the concept of independent events complex (Nabbout-Cheiban, 2017). The view that understanding the concepts communicated is the pivot upon which mathematics depends (Bhuvaneswari & Umamaheswari, 2020; Quinnell, 2019) underscores the importance of language in mathematics. For these reasons, studies of mathematical language may serve to understand mathematics and inform mathematics instruction.

3. Literature Review

3.1 Mathematics as Language

Mathematical language involves specific symbols, numerals, words, and their combinations in diagrams, graphs, and numerical expressions. Mathematical language may include words (Hornburg et al., 2018) such as more, less, take away, and most in numeracy; odd, even, and prime in numbers; mean, mode, and median in statistics; log and ln in logarithms; product, sum, difference, quotient, and remainder in operations; perimeter, area, radius, diameter, and chord in geometry; and sine, cosine, tangent, hypotenuse in trigonometry. Some of these words are used in general language and may therefore have different meanings in mathematical language. Thus, they must be relearned within the mathematical register. Their meanings may also differ across topics in mathematics, such as the median, which is used in statistics but not in geometry (Thompson & Chappell, 2007). When these words, phrases, symbols, and their syntax are used to communicate mathematics concepts, they are referred to as the language of mathematics.

The arrangement of these mathematical symbols, numerals, and words forms a special syntax that carries specific meanings. The meanings may be represented in multiple ways, such as graphs, diagrams, word problems, or algebraic expressions. This may sometimes be done pedagogically to aid learning. For example, diagrams and multiple representations are generally considered positive influences on learning (Batanero & Álvarez-Arroyo, 2024). From a syntactic point of view, for example, learners need to know that $3(x + 5) = y$ remains correct when written as $x + 5 = y/3$ (Leibowitz, 2016). Also, numerals such as 1 and 2 are values in terms of quantity that differentiate them (Schmitt et al., 2019). Thus, learning mathematics involves learning the language of mathematics, which is often specific to mathematics and, even more so, topic-specific (Hager, 2018). Through communication, such as writing and explaining, teachers can identify students' difficulties with the content (Sepeng & Madzorera, 2014). Experience shows that teachers tend to help learners understand the meaning of the mathematical language before focusing on mathematics itself. Also, Arias de Sanchez (2022) points out that learning the language of mathematics is part of learning mathematics. This suggests there may be a relationship between mastery of the content and the ability to communicate it. Various studies (Makonye, 2019; Sepeng & Madzorera, 2014) have examined English or mathematics language competence in relation to mathematics performance. The general conclusion is that there is a relationship between the ability to express oneself in English or in mathematical language and performance in mathematics (Jourdain & Sharma, 2016).

The literature suggests a close relationship between what happens in the common language and the formal mathematical language, requiring special attention to register that construes specific mathematical meanings (Jourdain & Sharma, 2016; Makonye, 2019). This view suggests that some registers carry special meaning in different contexts, and learners who come to the class with prior knowledge from a different context may be confused when their use is not clarified in a new context (Yanik, 2011). This assertion follows from the view that some words used in a common language may have precise meanings in other contexts, such as in normative mathematical conceptualisation (Yanik, 2011). However, mathematical language becomes more foreign when symbols are used to convey actions or expressions that act as complete sentences or when everyday words like table, factor, round, and eliminate are used in mathematics classrooms.

This study is situated in Tanzania, where subject-specific language is under-researched. In Tanzania, performance in mathematics has not only remained low (Kyaruzi et al., 2019; Kisakali & Kuznetsov, 2015), but has also affected boys and girls differently (Njiku, 2019). Boys and girls have been recorded to perform differently in mathematics, leading to gender gaps attributed to cultural factors (Mazana et al., 2020) and limited inclusive language. Previous research has shown that boys perform better than girls in mathematics (Mazana et al., 2020). These studies did not discuss gender differences in mathematical language. This study intends to examine this topic further by comparing the performance of boys and girls, including their mathematical language skills, especially in probability.

3.2 Mathematical Language Skills and Performance

The ability to express oneself in mathematics goes beyond the register of any common language, such as the English language. It can be done with minimal understanding of such a common language. Communication in mathematics involves mathematics-specialised language. Mathematical language is used to communicate mathematically and carries conceptual knowledge in mathematics. The language of mathematics is, in fact, concept-based, and without it, mathematics cannot be communicated. This area has received little attention in the literature. For example, Njiku (2024) points out that, despite some attention to vocabulary, little has been done to address ambiguities, double or multiple meanings, and word problems in mathematics. Consequently, the current study examines the level of mathematical language students possess and whether it relates to their mathematics performance.

With a focus on the relationship between ordinary and mathematical language,

Jourdain and Sharma (2016) argue that, for success in mathematics, students must learn the ordinary-language register and its specific meanings in mathematics. Thompson and Chappell (2007) add that discussing mathematical concepts with colleagues leads to a deeper understanding than merely solving mathematical problems. However, the language in use may have different influences on the difficulties. Njiku (2024) provides an example that the number 12 is called twelve, a name that seems to have no relationship with other numbers, as may be the case in Chinese, where 12 is ten and two. This is the case in some languages like Kiswahili, the native language of most participants in this study. Although it may be accepted that mathematics learning occurs in a cultural space (Makonye, 2019), 12, for example, communicates the quantity it represents to people regardless of their language. For example, the place values of 1 and 2 are known, and when performing operations, these values must be treated consistently irrespective of the language in use. This mathematical syntax does not change meaning or quantity, regardless of the form or nature of the language (Leibowitz, 2016). The 12 is a numerical representation of a quantity, and its meaning is consistently conveyed across cultures. Moreover, the positions of the numerals communicate yet another thing – the value. And mathematical syntax does not change across cultures.

When mathematical language is viewed from a non-symbolic perspective, it may suggest a yet different debate. For example, working on numeracy with young children, Fuchs et al. (2010) and Hornburg et al. (2018) conclude that mathematical language, including vocabulary suggested by Purpura and Logan (2015), such as more, less, take away, fewest, most, near, and far have no relationship with formal addition (such as in 1 + 2). Evidence from this line of thinking shows that learners can confidently perform addition with relevant mathematical expressions, such as 1 + 2. However, they are unable to succeed in solving story problems bearing the same concept (Mayer & Hegarty, 1996). On the contrary, it is also viewed that mathematical language can significantly predict the execution of mathematical problems (Schmitt et al., 2019). Given these divergent views, in this paper, we consider numerals such as 1, 2, and 3 and signs such as +, – , and = as a mathematical language register. Hence, they carry a specific meaning. When performing operations, one must know what 1 or 2 represents in terms of quantities and be able to interpret the expression 1 + 2 mathematically, appropriately use the sign =, and correctly communicate the answer 3 in a numerical form. Language use can potentially affect the learning of the subject matter (Yanik, 2011). This paper investigates how specific mathematical language influences students' performance, as the literature (Hornburg et al., 2018; Mayer & Hegarty, 1996)

indicates that some difficulties stem from an inability to comprehend problems rather than from carrying out procedures.

At the early stages of elementary education, students' mathematical language skills are naïve and primarily related to everyday, familiar experiences (Simpson & Cole, 2015). Students aged 7-8 may use probability language accurately, especially when explaining the likelihood of chance events, but find it hard to explain impossibility and certainty, even by 11-12-year-old students (Groth et al., 2020; Kazak & Leavy, 2018). Students whose home language is not English tend to have limited skills in interpreting mathematical problems compared to native English speakers (Christiansen & Aungamuthu, 2012). Teachers are recommended to bridge these gaps between everyday language and formal mathematical language to avoid confusion. Studies examining probability language have focused solely on vocabulary (Findley et al., 2021; Groth et al., 2016). In this study, we extend the discussion to other elements of probability language, including symbols (writing given sentences/phrases in symbols), wording (writing given symbols in words), and interpretation (explaining what a mathematical statement means).

3.3 Probability Language

In probability, the focus of this paper, multiple studies (Nabbout-Cheiban, 2017; Sharma, 2014; Teigen et al., 2014; Wagner et al., 2015) have been conducted. Teigen et al. (2014) show that probability language is common, as in explanations of the possibility of rain tomorrow or of whether people's interest in a political wing will increase. In such cases, percentages may be used. Words such as certain, possible, most likely, random, mutually exclusive, and chance carry probability meaning. Gal (2005) identifies other terms such as randomness, chance, independence, predictability, and certainty. It follows that probability is part of everyday life and influences individual and collective decisions (Gal, 2005; Sharma, 2014). Probability language is a combination of words, symbols, and their syntax used to communicate concepts in probability topics. In the secondary school curriculum, a combination of symbols such as P(A) and P(A'), words like complement and union, and interpreting statements like the probability of an event is 1 carries this specific language of definite concepts in probability. Language is considered a window to understanding probability (Wagner et al., 2015). Knowledge of probability is essential for interpreting and critically evaluating data-based information (Sharma, 2014).

Probability literacy includes, among other domains, the language of probability. Probability language includes understanding words and phrases, as well as various ways to represent and communicate about the occurrence of events

(Kazak & Leavy, 2018). Whereas many studies on probability language are limited to vocabulary (Findley et al., 2021; Groth et al., 2020; Groth et al., 2016), mathematical language contains multiple elements, including written and oral language, symbols, and visual representations (Simpson & Cole, 2015). These elements are essential for the appropriate interpretation of mathematical problems. Although understanding probability is seen as crucial, and mathematical language influences performance, little is known about the relationship between these two: probability performance and probability language skills. In this paper, we examine students' probability language skills and their influence on students' performance on the topic. To do this, we assess three four elements, vocabulary, wording, symbols and interpretations to address the following questions:

i) What mathematical language skills do students possess in probability?

ii) Do students' probability language skills differ by gender?

iii) What is the relationship between students' mathematical language skills and performance in probability?

4. Methodology

4.1 Study Design and Participants

Situated in the classroom where one of the authors was teaching, the study was designed to examine the relationship between students' mathematical language skills and their performance in mathematics. Specifically, the study employed a quantitative approach and adopted a correlational research design to examine the relationship between probability language skills and performance in probability. The study uses data about students' gender, mathematical language skills, and achievement in probability. The study involved 49 secondary school students (18 females and 31 males). In the context of the study, these were fourth-year secondary students whose syllabus covered probability. Their age range was 14-18 years ($M = 16.49$, $SD = 0.74$). Because the study assessed knowledge of probability and its language, the students selected for the study had completed the curriculum topic on probability. To reduce potential biases, data analysis was conducted by the other author. Additionally, participants were assured of anonymity, and consent was obtained before their participation.

4.2 Instruments and Data Analysis

The researchers developed the data collection tools. The development was especially necessary for the language part since no existing tools from the Tanzanian context assessed the language of mathematics. To assess students'

probability language skills, we used items distributed across four dimensions: symbols: 3 items (for example, write in symbols: Probability of A intersection B is equal to two out of five), wording: 2 items (for example, write in words: P(A')), interpretations: 2 items (for example, What does it mean by saying that the probability of an event is 1?), and vocabulary: 2 items (for example, What it means by impossible event?). We also used five items to assess students' achievement in probability. Content validity was ensured by designing items on the topic under investigation (Ary et al., 2010) – probability – and testing them only with students who had learned the topic. Two experts with experience in teaching the topic reviewed the items for relevance. The split-half reliability by the Spearman-Brown formula was used to calculate the coefficient where:

= estimated reliability of the test

= person r correlation between the two halves of the test.

The language test had the reliability of r_{xx} = .673, and the achievement test had the reliability of r_{xx} = .722. Although the coefficients were low and acceptable rather than good, we proceeded with the study because this might have been partly due to the test length (Ary et al., 2010), given that the study was classroom-based. Although the number of achievement test items was five, they were considered sufficient because a single mathematical problem may be solved through several stages that require knowledge from multiple concepts and adequate time.

Research questions guided the data analysis. Descriptive statistics (mean for central tendency and standard deviation for dispersion) were used to address the first research question on students' mathematical language skills. The second research question was addressed using descriptive statistics (mean and standard deviation) and the *t*-test. The Pearson correlation was used to examine the relationships among mathematical language skills and achievement. The study did not use effect sizes to complement the *p*-values for the *t*-test since non-significant differences were observed.

5. Findings

5.1 Students' Level of Probability Language Skills

To address the first research question, descriptive statistics were used to analyse students' performance in the language skills test. Findings are presented in Table 1, by specific language dimension and overall, using means and standard deviations. Each dimension was scored out of 100 for consistency.

Table 1
Students' Performance on Probability Language Skill Test

	Mean	Standard Deviation
Symbols	80.44	6.33
Wording	76.36	7.23
Interpretation	41.84	4.67
Vocabulary	83.67	6.45
Overall language	70.58	18.13

Figure 1: *Students' Scores in Probability Language Skills*

Overall performance in the probability language test was 70.6%. Similarly, data show that students performed highly in the dimensions of symbols, wording, and vocabulary, with scores of 80.4%, 76.4%, and 83.7%, respectively. However, the score on students' ability to interpret probability concepts was only 41.8%. Figure 1 shows language scores ranging from 21 to 88, with the majority (31) of students scoring above the average of 70.58. This suggests that students who scored below the mean, although few (18), greatly affected the overall mean. The median score was 75, the minimum score was 21, and the maximum was 87.5.

5.2 Probability Language Skills by Gender

Further analysis examined how students' performance levels in probability language skills were distributed based on gender. This comparison used both descriptive and inferential statistics. In this analysis, achievement test scores (out of 100) were also included. Table 2 below summarises the findings using mean scores and standard deviation.

Table 2
Students' Performance by Gender

Gender	Symbols	Wording	Interpretation	Vocabulary	Language	Achievement
Female	78.75(2.63)	79(3.01)	43.58(2.05)	85.5(3.17)	71.71(7.31)	42.56(7.82)
Male	83.33(3.69)	71.75(4.19)	38.92(2.57)	80.58(3.01)	68.65(10.87)	40.00(10.63)

Note: Language scores are the average scores of symbols, wording, interpretation, and vocabulary.

Findings show that females outperformed males in the achievement test, the overall probability language skill test, and three of the four dimensions of the probability language skill test. These differences were further investigated using

an independent samples *t*-test. The findings from this analysis are summarised in Table 3.

Table 3

t-test Results for Probability Language and Achievement Tests Between Female and Male Students

	t-statistic	*p*-value
Symbols	-0.605	0.548
Wording	0.775	0.445
Interpretation	0.840	0.405
Vocabulary	0.641	0.525
Language	0.512	0.613
Achievement	0.390	0.699

From Table 3, it may be observed that none of the *p*-values was less than 0.5. This suggests that none were statistically significant, despite differences in mean scores between females and males.

5.3 Relationship Between Probability Language Skills and Performance in Probability

The bivariate Pearson correlation coefficient was used to address the third research question, which examined the relationship between language skills and performance. Students' scores in the probability language skill test and the achievement test were correlated, and the findings are summarised in Table 4.

Table 4

Correlation Between Probability Language Skills and Achievement Test Scores

	Symbols	**Wording**	**Interpretation**	**Vocabulary**	**Language**	**Achievement**
Symbols	1.000					
Wording	.304*	1.000				
Interpretation	.610**	.407**	1.000			
Vocabulary	.298*	.240	.583**	1.000		
Language	.733**	.695**	.840**	.705**	1.000	
Achievement	.504**	.244	.486**	.376**	.532**	1.000

Note. $^{**}p < .01$, $^{*}p < .05$

The findings from correlation analysis show that scores from the probability language skills test correlated positively with performance in the probability achievement test, r (49) = 0.532, $p < .01$. This means that the coefficient of determination $r^2 = .283$, indicating that about 28.3% of the variability in

the achievement test may be explained by the probability language skills. The relationship between the specific probability language dimensions and achievement test performance was positive and significant except for wording. This suggests that the ability to turn probability symbolism into words did not relate to performance in probability.

6. Discussion

The study assessed secondary school students' probability language skills. Language skills enable students to understand and communicate mathematically. Limited ability to communicate mathematically may consequently affect their learning of the topic and performance (Nacarato and Grando, 2014), although such an analysis was not conducted in the study. Findings show that students scored high in language skills in general. It is worth noting that the inquiry into the relationship between language and thought is historical and is evident in Vygotsky's (1986) classical work, Thought and Language. However, when specific language dimensions were examined, students scored high in symbols, wording, and vocabulary but low in interpreting mathematical meanings. Similar findings are reported by Kotsopoulos (2007), who argues that interpretation requires an understanding of mathematical register and the context in which it is used, and that learners find it challenging to associate everyday register with mathematical register. Unlike the current study, in which vocabulary was scored highly, Nacarato and Grando (2014) report challenges with vocabulary; their findings align with this study's findings regarding how such vocabulary is used in real contexts, as shown in the interpretations students provided, which differed from the appropriate mathematical registers. In this study, for example, students were told to explain the meaning of "the probability of an event is zero" and "the probability of an event is one". Students might have needed to use words such as 'impossible' or 'sure event' to provide such explanations. However, as Lesser et al. (2016) noted, this would be difficult for learners whose language of instruction is not their first language. Furthermore, the limited ability to interpret mathematical meanings suggests the need to integrate mathematical language in instruction since, as Arias de Sanchez (2022) argues, language does not exist in isolation.

Females and males may have different language abilities, which may also result from classroom or out-of-class interaction. Gender differences in relation to mathematical language have received little attention in the literature. For example, although Bhuvaneswari and Umamaheswari (2020) collected data about participants' gender, their analysis lacks this variable. Kotsopoulos (2007) shows that interaction between students leads to register interference, which may

include peer-appropriate language but differs from appropriate mathematical registers. No differences are expected between females and males when such interaction is not gender-based. The findings show no significant difference between females and males in probability language skills. Nevertheless, the descriptive statistics showed that females performed better than males, except in one dimension.

Although the descriptive statistics showed that females generally had better language skills and outperformed males on the achievement test, the difference was not statistically significant. In previous studies, males have usually been reported to do better in mathematics than females (Njiku, 2019). Sampling and sample size are likely contributing factors to these findings. Unlike in this study, when such differences become significant, teachers may need to facilitate mathematical discussions and conversations during learning sessions to bridge gaps that may exist between females and males. Teachers may need to do so to facilitate mathematical discussions and conversations during learning sessions. The findings show that language skills and achievement are positively correlated. Limited studies have examined the topic in relation to performance. For example, focusing on English rather than mathematical language, Tanko et al. (2022) examined its relationship with mathematics performance and found a positive, significant relationship.

However, various studies have framed their arguments around the relationship between mathematical language and learning or difficulties in learning. For example, Nacarato and Grando (2014) argue that students' familiarity with probabilistic language provides them with a linguistic repertoire that they may use to justify their mathematical solutions. Similarly, Quinnell (2019) associates understanding mathematical language with understanding mathematical concepts. However, in this study, one dimension – wording – had no significant correlation. These results were surprising, as the wording seems to be deeper than simple vocabulary, involving articulation in writing (Chin et al., 2022). This is probably because turning symbols into words requires them to use mathematical registers that they unfortunately use in everyday life, but in a different context (Nacarato & Grando, 2014). A similar observation is made by Nabbout-Cheiban (2017), who found that students encountered difficulties understanding the concept of independent events, and the author argues that this was probably due to the meaning of the word 'independent' as used in daily life language. Also, similar difficulties in probability are reported among teachers and are also partly associated with language (Molnar, 2016).

7. Limitations of the Study

Since the study was contextualised in the classroom, the context may differ from other schools. Thus, it is difficult to generalise; where needed, it should be done cautiously. Also, since the study adopted a quantitative approach, it did not provide qualitative perspectives. Further investigations are recommended to adopt mixed-methods approaches to provide a more comprehensive picture of the nature of the language students use in mathematics. Furthermore, the study used a correlational design with a small sample size, which does not establish causal claims. Further investigations may address the limitations of the design by adopting designs that establish a causal relationship between probability language skills and achievement in probability. Furthermore, the achievement test consisted of only five mathematical problems. The assessment tool for probability achievement can be improved by increasing the number of items that span various concepts of the topic, while accounting for other factors such as time, item difficulty, and the problem-solving demands of the items.

8. Conclusion

The study found a relationship between probability language skills and achievement, where students demonstrated fairly good language competencies. The relationship may suggest that as students become familiar with mathematical registers and their use in mathematical contexts, they also develop the ability to understand and solve mathematical problems. Although the relationship examined in this study is not causal, a positive association was observed, which warrants further causal and qualitative investigation. To this end, teachers may need to dedicate more time, through action research, to exploring how mathematical language can be appropriately adopted in the classroom. This may also include practising dialogue in mathematics classrooms in order to develop subject-specific language competencies. To do this, we argue that teachers must clearly understand students' needs in mathematical language and actively engage students in articulating the elements of mathematical language that may be used across various mathematical problems.

References

Arias de Sanchez, G. (2022). Subtract? That's a math word! Unpacking teachers' language choices in preschool and kindergarten classrooms. *European Journal of Science and Mathematics Education, 10*(3), 366-379. https://doi.org/10.30935/scimath/11988

Ary, D., Jacobs, L. C., & Sorensen, C. K. (2010). *Introduction to research in education.* Wadsworth, Cengage Learning. https://shorturl.at/beFNP

Aubrey, K., & Riley, A. (2022). *Understanding and Using Educational Theories.* Sage. https://www.torrossa.com/it/resources/an/5282296

Batanero, C., & Álvarez-Arroyo, R. (2024). Teaching and learning of probability. *ZDM–Mathematics Education*, *56*(1), 5-17. https://doi.org/10.1007/s11858-023-01511-5

Bhuvaneswari, R., & Umamaheswari, S. (2020). Integrating vocabulary instruction into math classroom. *Advances in Mathematics: Scientific Journal, 9*(4), 1541-1551. https://doi.org/10.37418/amsj.9.4.8

Chin, K. E., Jiew, F. F., & Tall, D. (2022). The Articulation Principle for making long-term sense of mathematical expressions by how they are spoken and heard: Two case studies. *The Mathematics Enthusiast, 19*(2), 657-676. https://doi.org/10.54870/1551-3440.1567

Christiansen, I., & Aungamuthu, Y. (2012). Language issues, ''misconceptions''and confusion: A qualitative analysis of KZN grade 6 learners' responses on a mathematics test. *Education as Change*, *16*(1), 51-67. https://doi.org/10.1080/16823206.2012.691713

Findley, K., Whitacre, I., & Atabaş, Ş. (2021). What's in a name? Leveraging students' informal vocabulary in probability. *Teaching Statistics*, *43*(2), 72-78. https://doi.org/10.1111/test.12250

Fuchs, L. S., Geary, L. C., Compton, D. L., Fuchs, D., Hamlett, C. L., Seethaler, P. M., . . . Schatschneider, C. (2010). Do different types of school mathematics development depend on different constellations of numerical versus general cognitive abilities? *Developmental Psychology, 46*(6), 1731–1746. https://doi.org/10.1037/a0020662

Gal, I. (2005). Towards" probability literacy" for all citizens: Building blocks and instructional dilemmas. In G. A. Jones, *Exploring probability in school: Challenges for teaching and learning* (pp. 39-63). Springer US. https://doi.org/10.1007/0-387-24530-8_3

Groth, R. E., Bergner, J. A., & Austin, J. W. (2020). Dimensions of learning probability vocabulary. *Journal for Research in Mathematics Education*, *51*(1), 75-104. https://doi.org/10.5951/jresematheduc.2019.0008

Groth, R. E., Butler, J., & Nelson, D. (2016). Overcoming challenges in learning probability vocabulary. *Teaching Statistics*, *38*(3), 102-107. https://doi.org/10.1111/test.12109

Hager, G. (2018). Disciplinary Literacy in the Mathematics Classroom. Mathematics Education Research Group of Australasia. *Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA*) (pp. 361-368). https://eric.ed.gov/?id=ED592424

Hornburg, C. B., Schmitt, S. A., & Purpura, D. J. (2018). Relations between preschoolers' mathematical language understanding and specific numeracy skills. *Journal of Experimental Child Psychology, 176*, 84–100. https://doi.org/10.1016/j.jecp.2018.07.005

Jourdain, L., & Sharma, S. (2016). Language challenges in mathematics education: A literature review. *Waikato Journal of Education, 21*(2), 43-56. https://eric.ed.gov/?id=EJ1233433

Kazak, S., & Leavy, A. M. (2018). Emergent reasoning about uncertainty in primary school children with a focus on subjective probability. In *Statistics in early childhood and primary education: Supporting early statistical and probabilistic thinking* (pp. 37-54). Springer. https://doi.org/10.1007/978-981-13-1044-7_3

Kisakali, J., & Kuznetsov, D. (2015). Modeling factors influencing mathematics learning and performance in Tanzanian secondary schools. *Asian journal of mathematics and applications*. 2015. https://scienceasia.asia/files/244.pdf

Kotsopoulos, D. (2007). Mathematics discourse: It's like hearing a foreign language. *The Mathematics Teacher, 101*(4), 301-305. https://doi.org/10.5951/MT.101.4.0301

Kyaruzi, F., Strijbos, J.-W., Ufer, S., & Brown, G. T. (2019). Students' formative assessment perceptions, feedback use and mathematics performance in secondary schools in Tanzania. *Assessment in Education: Principles, Policy & Practice, 26*(3), 278-302. https://doi.org/10.1080/0969594X.2019.1593103

Leibowitz, D. (2016). Supporting mathematical literacy development: A case study of the syntax of introductory algebra. *Interdisciplinary Undergraduate Research Journal, 2*(1), 7-13. https://journals.uic.edu/ojs/index.php/IURJ/article/view/6668

Lesser, L. M., Wagler, A. E., & Salazar, B. (2016). Flipping between languages? An exploratory analysis of the usage by Spanish-speaking English language learner tertiary students of a bilingual probability applet. *Statistics education research journal, 15*(2), 145-168. https://doi.org/10.52041/serj.v15i2.245

Makonye, J. P. (2019). The effect of translanguaging in teaching the Grade 6 topics of perimeter and area in rural schools. *Southern African Linguistics and Applied Language Studies, 37*(3), 221-231. https://doi.org/10.2989/16073614.2019.1671880

Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R. J. Sternberg, & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 45-70). Routledge. https://doi.org/10.4324/9780203053270

Mazana, M. Y., Montero, C. S., & Casmir, R. O. (2020). Assessing students' performance in mathematics in Tanzania: The teacher's perspective. *International Electronic Journal of Mathematics Education, 15*(3), em0589. https://doi.org/10.29333/iejme/7994

Molnar, A. (2016). High school mathematics teachers' understanding of independent events. *In 13th International Congress on Mathematical Education. 25*, (pp. 1-7). ICME. http://iase-web. org/documents/papers/icme13/ICME13_L9_Molnar. pdf

Nabbout-Cheiban, M. (2017). Intuitive thinking and misconceptions of independent events: A case study of US and French pre-service teachers. *International Journal of Research in Undergraduate Mathematics Education, 3*(2), 255-282. https://doi.org/10.1007/s40753-016-0038-x

Nacarato, A. M., & Grando, R. C. (2014). The role of language in building probabilistic thinking. *Statistics Education Research Journal, 13*(2), 93-103. https://doi.org/10.52041/serj.v13i2.283

Njiku, J. (2019). Mathematics performance across gender and who owns a school. *Huria: Journal of the Open University of Tanzania*, *26*(1). https://doi.org/10.61538/huria.v26i1.691

Njiku, J. (2024). Language Barriers in Mathematics: Learning Challenges When Global Languages Replace Native Instruction. *International Journal of Innovation in Science and Mathematics Education*, *32*(6). https://doi.org/10.30722/32.06.001

Purpura, D. J., & Logan, J. A. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology, 51*(12), 1717–1724. https://doi.org/10.1037/dev0000055

Quinnell, L. (2019). Placing an emphasis on mathematical language as a key to unlocking mathematical understanding. *Australian Journal of Middle Schooling, 19*(1), 4-13. https://www.adolescentsuccess.org.au/resources/Documents/Vocabulary%20introduction%20Lorna%20Quinell.pdf

Schmitt, S. A., Purpura, D. J., & Elicker, J. G. (2019). Predictive links among vocabulary, mathematical language, and executive functioning in preschoolers. *Journal of Experimental Child Psychology, 180*, 55-68. https://doi.org/10.1016/j.jecp.2018.12.005

Sepeng, P., & Madzorera, A. (2014). Sources of difficulty in comprehending and solving mathematical word problems. *International Journal of Educational Sciences, 6*(2), 217-225. https://doi.org/10.1080/09751122.2014.11890134

Sharma, S. (2014). Cultural influences in probabilistic thinking. In E. Chernoff, & B. Sriraman (Eds.), *Probabilistic Thinking, Advances in Mathematics Education* (pp. 657-681). Springer. https://doi.org/10.1007/978-94-007-7155-0_35

Simpson, A., & Cole, M. W. (2015). More than words: A literature review of language of mathematics research. *Educational Review*, *67*(3), 369-384. http://dx.doi.org/10.1080/00131911.2014.971714

Tanko, I. Z., Yushau, B., & Arhyel, I. (2022). Relationship between mathematics performance and English language proficiency among senior secondary school students in Bauchi metropolis, Nigeria. *Galaxy International Interdisciplinary Research Journal, 10*(5), 509-516. https://giirj.com/index.php/giirj/article/view/3206

Teigen, K. H., Juanchich, M., & Filkuková, P. (2014). Verbal probabilities: An alternative approach. *Quarterly Journal of Experimental Psychology, 67*(1), 124-146. https://doi.org/10.1080/17470218.2013.793731

Thompson, D. R., & Chappell, M. F. (2007). Communication and representation as elements in mathematical literacy. *Reading & Writing Quarterly: Overcoming Learning Difficulties, 23*(2), 179-196. http://dx.doi.org/10.1080/10573560601158495

Vygotsky, L. (1986). *Thought and Language (A. Kozulin, trans.)*. In. Cambridge, MA: Massachusetts Institute of Technology Press.

Wagner, D., Dicks, J., & Kristmanson, P. (2015). Students' language repertoires for prediction. *The Mathematics Enthusiast, 12*(1), 246-261. https://unbscholar.lib.unb.ca/handle/1882/22406

Yanik, B. H. (2011). Prospective middle school mathematics teachers' preconceptions of geometric translations. *Educational Studies in Mathematics, 78*(2), 231-260. https://doi.org/10.1007/s10649-011-9324-3